University of Groningen

Examination - 2020

Date set: 16th June, 2020	Candidate Name:	Setter: Anupam Mazumdar
Final Examination	NAKF2-11/Quantum Physics2	Total Marks = 1+(...)/5

Marks

Follow these instructions:

(1) Kindly sign the declaration from the Board of Examiners before your exam. The time of your submission will be automatically recorded in NESTOR.

(2) Your answers should be legible (if it is not readable, it will not be marked at all).

(3) You are allowed to follow any books, lecture notes.

(4) Marks will be given on understanding of the topic, so physical understanding along with mathematical results, will lead to higher marks.

(5) The deadline for submission of your exam paper is: 16 th June, 12:00 hrs. Scan your answer paper and upload it in One Single File in your respective EXAM TENTAMEN folder in Nestor.

(6) Those who require extra time, you will be allowed to take extra time. Note that your timing will be recorded by Nestor.

Declaration of the Board of Examiners

The Board of Examiners has allowed the conversion of your exam into a take-home exam. This conversion comes with additional provisions. Here are the provisions that are relevant to you sitting the exam:

1. You are required to sign the attached pledge, swearing that your work has been completed autonomously and using only the tools and aids that the examiner has allowed you to use.

2. Attempts at cheating, fraud or plagiarism will be seen as attempts to take advantage of the Corona crisis and will be dealt with very harshly by the board of examiners.

3. The board of examiners grants your examiner the right to conduct a random sampling. If you are selected for this sample, you may be required to conduct a discussion (digitally, using audio and video) in which you are asked to explain and/or rephrase (some of) the answers you submitted for the take-home exam.

I .... (enter your name and student number here) have completed this exam myself and without help from others unless expressly allowed by my lecturer. I have come up with these answers myself. I understand that my fellow students and my lecturers are all doing their best to do their work as well as possible under the unusual circumstances of the Corona pandemic, and that any attempt by myself or my fellow students to use these circumstances to get away with cheating would be undermining those efforts and the necessary trust that this moment calls for.

C1. **(a) (i) Define the Spin operators, $\mathbf{S_x}, \mathbf{S_y}, \mathbf{S_z}$, in terms of the Pauli matrices.** [1]

(ii) Show that the following fundamental commutation relationship holds:

$$[S_x, S_y] = i\hbar S_z$$

[2]

(b) Consider the state $|j_1 j_2 j m\rangle$, which is a common eigenstate of the angular momentum operators $\mathbf{J_1^2}, \mathbf{J_2^2}, \mathbf{J^2}$ and j_z, where $\mathbf{J} = \mathbf{J_1} + \mathbf{J_2}$.

(i) What are the values j can take? [1]

(ii) Find the eigenvalue of the operator $\mathbf{J} \cdot \mathbf{J_1}$ [2]

C2. **Let us study the effect of an external uniform electric field, which is directed along the positive z axis, $\mathbf{E} = E\vec{k}$, on the ground state of a hydrogen atom. Let us ignore the spin degrees of freedom.**

(i) Find the total Hamiltonian of the hydrogen atom including the unperturbed and the perturbed hamiltonian. [2]

(ii) Find the first order correction to energy eigenvalue $E_{100}^{(1)}$, where the subscript 100 corresponds to the ground state of the Hydrogen atom. [4]

(iii) Write down the expression for the second order correction to the energy eigenvalue $E_{100}^{(2)}$ without evaluating the integration. [2]

(iv) Let us assume that $E_{100}^{(0)} - E_{nlm}^{(0)} \leq E_{100}^{0} - E_{200}^{(0)}$ and assume $E_{100}^{(0)} - E_{nlm}^{(0)}$ to be constant, then evaluate $E_{100}^{(2)}$. [8]

C3. **Consider a one-dimensional harmonic oscillator in x direction. Use the variational method to estimate the following:**

(i) Find the trial wave function, $\psi_1(B, \alpha, x)$, where (B, α) are two constants, x is the variable. We know that the wavefunction is an odd function of x, and vanishes at $x \to \pm\infty$. [3]

(ii) Normalise the wavefunction and determine one of the constants. [3]

Hint: Use $\int_{-\infty}^{\infty} x^{2n} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha}\ 1.3.5...(2n-1)/(2\alpha)^n$

(iii) Write down the Hamiltonian for a one-dimensional harmonic oscillator. [1]

(iV) Find the energy of the first excited state by minimising the energy condition. [4]

(v) By substituting the values of (B, α) find the wavefunction $\psi_1(x)$. [3]

C4. In a time-dependent perturbation theory the transition probability for $|\psi_i\rangle \to |\psi_f\rangle$, with $i \neq f$ is given at the first order in terms of the time-dependent perturbation $V(t)$, as

$$P_{if}(t) = \left| -\frac{i}{\hbar} \int_0^t \langle \psi_f | \hat{V}(t') | \psi_i \rangle e^{\omega_{fi} t'} dt' \right|^2 .$$

(i) Assume that $\hat{V}(t)$ vanishes at the limits (when it is switched on at $\hat{V}(t) = 0$ and off at time $\hat{V}(t) = 0$). Show that the above expression for the transition probability can then be expressed with the help of integration by parts:

$$P_{if}(t) = \frac{1}{(\hbar \omega_{fi})^2} \left| \int_0^t e^{i\omega_{fi} t'} \left(\frac{\partial}{\partial t'} \langle \psi_f | \hat{V}(t') | \psi_i \rangle \right) dt' \right|^2$$

[4]

(ii) The adiabatic limit is when $\hat{V}(t)$ changes very smoothly, means that it will change very little in the time interval $0 \leq t' \leq t$. Show that the transition probability can be approximated by:

$$P_{if}(t) \approx \frac{4}{\hbar^2 \omega_{fi}^4} \left| \frac{\partial}{\partial t} \langle \psi_f | \hat{V}(t) | \psi_i \rangle \right|^2 \sin^2 \left(\frac{\omega_{fi} t}{2} \right)$$

[4]

(iii) From this last step, argue under what conditions this transition probability will become 0. This limit is known as the adiabatic limit. [1]